

Math Together™ is a research-based intervention that aligns with NCTM standards, teaching the five content strands (numeration, measurement, algebra, geometry and data analysis) through the defined processes. Its interdisciplinary approach combines mathematics, social studies and language arts, placing problem solving in context and enhancing reading and writing skills. Using a structured tutoring format, Math Together incorporates the strategies most essential to increasing student achievement, as outlined by the National Assessment of Education Progress:

- *Relating mathematics to the real-world experiences of young people;*
- *Working cooperatively to solve problems;*
- *Exploring mathematics concepts with hands-on materials;*
- *Using calculators; and*
- *Constructing one's own mathematical knowledge.*

Tutors learn and practice mathematics strategies during pre-lesson training sessions with their coordinator. Using scripted lessons, tutors then model for tutees by demonstrating or thinking aloud during the tutorial. Tutees learn how to construct their own mathematical knowledge, how to communicate mathematically and how to justify their reasoning. Problem-solving strategies are used repeatedly and consistently until the tutees can apply them independently, across curriculum and in real-life situations.

Strategy	Rationale	Cited research
<p>Warm-up chat: Tutor initiates a short conversation with tutee. A close tutor/tutee bond is at the heart of the cooperative learning that takes place throughout each lesson.</p> <p>Tutor and tutee learn to communicate about mathematics, using appropriate vocabulary.</p>	<p><i>Motivation and relationship building:</i> Working with a partner promotes more positive attitudes and high achievement.</p> <p>Multi-age interactions provide an environment in which close relationships allow for the development of mutual trust and understanding. These relationships and the scaffolding of growth opportunities provided by a multi-age peer group make a significant contribution to academic growth.</p> <p>Building a community of learners in a risk-free environment is one of the seven best learning strategies for enhancing mathematics education.</p> <p>Students should be engaged in the construction of mathematical understanding through the use of group work, open discussions, presentations and verbalization of mathematical ideas.</p> <p>All aspects of motivation are enhanced when students feel like they can control their own learning because of their development of a personal toolbox of strategies.</p> <p>Studies show using small groups of students working together in mathematics increases achievement over control groups in traditional classrooms.</p>	<p>Johnson and Johnson 1986</p> <p>Kinsey 2000</p> <p>Bickmore-Brand 1993</p> <p>Mathematical Sciences Education Board 1989</p> <p>Swartz 2002</p> <p>Davidson 1985</p>

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<p>Warm-up chat (continued)</p>	<p>Many students may feel more at ease, and thus can concentrate better on the subject matter, with a peer tutor rather than a professional teacher or consultant. Peer tutors help themselves increase their own understanding of subject matter they tutor students in/on, which boosts confidence and can carry over to their desire to learn other subjects.</p> <p>Peers are more sensitive than adult teachers to picking up on non-verbal cues that tutees may not understand what a tutor is trying to communicate to them...a peer tutor may be able to more readily perceive difficulties a student being tutored may be having, and can then work to clear things up.</p> <p>Peer assisted learning, usually tutoring, could revolutionize (mathematical) practice. When kids are partnered up, one stronger student and one weaker student, achievement is always improved.</p> <p>Cooperative learning methods are effective in improving student achievement.</p>	<p>Ehly and Karsibm 1980</p> <p>Allen and Feldman 1996</p> <p>Gersten 2003</p> <p>Slavin 1990</p>
<p>Launch Activity Most lessons start with a simple, hands-on activity that introduces the concepts or vocabulary to be covered.</p>	<p>Hands-on tools help children begin to visualize and construct for themselves the patterns of our number system. These skills enable them to understand computation and develop efficient strategies for learning facts.</p> <p>Mathematics is no longer an instruction process in which the teacher transmits information to students; instead we must replace this format with one in which the students construct their own understandings.</p> <p>Allowing students to use blackboards, bulletin boards and large floor space to demonstrate mathematic concepts can be enriching. Assignments and activities can be structured to include flexibility so students can utilize perceptual strengths (visual, auditory, kinesthetic or tactile).</p> <p>What counts is for kids to get their hands on the problem, either literally, by using manipulative or some other concrete model, or figuratively.</p> <p>For “effort” to occur: (1) the person must value the task and (2) the person must believe he or she can succeed at the task. Therefore, in an instructional situation, the learning task needs to be presented in a way that is engaging and meaningful, and in a way that promotes positive expectations for the successful achievement of learning objectives.</p>	<p>Hamilton-Weiler 1988</p> <p>D’Ambrosio, Johnson and Hobbs 2002</p> <p>Thompson and Mascazine 1997</p> <p>Herrera and Ozgun-Koca 2003</p> <p>Porter and Lawler 1968</p>

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<p>Before-reading activities: Tutee predicts content using picture and title to draw the reader into the story and activate background knowledge.</p>	<p><i>Predicting:</i> Asking students to make predictions entices them to read the story more carefully and to use context clues more effectively as they work through the process of confirming or rejecting their predictions. Students appear to remember events in the story better when the events discussed contribute to their prediction making.</p> <p><i>Activating prior knowledge:</i> We know that prior knowledge is an important step in the learning process. It is a major factor in comprehension: that is, making sense of our learning experiences. Brain-based research confirms the fact that the learning environment needs to provide a setting that incorporates stability and familiarity. It should be able to satisfy the mind's enormous curiosity and hunger for discovery, challenge, and novelty. Creating an opportunity to challenge our students to call on their collective experiences (prior knowledge) is essential. Through this process we move students from memorizing information to meaningful learning and begin the journey of connecting learning events rather than remembering bits and pieces. Prior knowledge is an essential element in this quest for making meaning.</p>	<p>Calderon 1999</p> <p>Christen and Murphy 1991</p>
<p>Shared Reading Tutor and tutee take turns reading from stories about key events in US history.</p>	<p><i>Reading-Math Connections:</i> Reading children's literature about mathematics needs more emphasis in the K-4 curriculum... children need opportunities to communicate and to "talk math" with their friends.</p> <p>Reading provides both context and motivation for math students. The use of fiction or non-fiction can create the context for discussion and set the stage for math skills.</p> <p>By cutting across multiple disciplines, students are encouraged to become active learners equipped with the analytical, interpretative and evaluative skills needed to solve real-life problems. Eliminating artificial barriers between subject areas gives students broader context for solving real-life problems.</p> <p>Linking mathematics to children's literature helps students connect mathematical ideas to their personal experiences, accommodates different learning styles and promotes critical thinking.</p> <p>Fourth-grade students use story, metaphor and language to develop mathematical thinking skills and strategies.</p>	<p>NCTM Position Paper 1989</p> <p>Balas 1997</p> <p>NCTM Position Statement 2003</p> <p>Murphy 2000</p> <p>Whitin and Whitin 2000</p>

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<p>Shared Reading (continued)</p>	<p>Encouraging mathematics students to talk, write, draw and enact texts provides a concrete way to construct and negotiate interpretations of what they read. In addition to helping students better understand the text read, acting on and acting out a text allow students to use that text as a springboard for sense-making and discussion of important mathematical ideas and issues about the nature of mathematics.</p> <p><i>Modeling:</i> By listening to good models of fluent reading, students learn how a reader’s voice can help written text make sense...Reading to children also increases their knowledge of the world, their vocabulary, their familiarity with written language (“book language”) and their interest in reading.</p> <p><i>Read aloud:</i> Through read alouds teachers demonstrate their thinking process when reading. These instructional demonstrations or think alouds are central to comprehension instruction.</p> <p>Reading aloud to children helps them grasp the structure of stories better, thereby enhancing their comprehension of texts and propelling them towards becoming independent readers.</p> <p><i>Practice reading:</i> Cross-age reading provides upper-grade youngsters with a legitimate reason for practicing for an oral reading performance.</p> <p>Reading to a younger child helps an older child develop positive reading behaviors.</p>	<p>Borasai et al 1998</p> <p>Center for the Improvement of Early Reading Achievement (CIERA) 2001</p> <p>Davey 1983</p> <p>Teale 1984</p> <p>Vacca, Vacca, and Gove 1991</p> <p>Labbo and Teale 1990</p>
<p>Praise and feedback Tutor encourages, corrects mistakes, praises tutee for genuine effort.</p> <p>Praise and correction techniques are scripted into each lesson.</p>	<p>Feedback is the most powerful single modification to enhance achievement.</p> <p>Timely feedback throughout the learning experience (“formative” as opposed to “summative” assessment) could drastically improve student achievement. The gains in achievement appear to be quite considerable...amongst the largest ever reported for educational interventions.</p> <p>A smile, praise, a hand on the shoulder and encouragement can have a significant effect on children’s behavior.</p> <p>It is important to have a classroom environment where students feel comfortable questioning, challenging, suspending judgment and demanding reasons and justification as they deal with mathematical and real-world content.</p> <p>The most successful tutors often have well-rehearsed scripts for</p>	<p>Hattie 1992</p> <p>Black and William 1998</p> <p>O’Leary and O’Leary 1997</p> <p>O’Daffer and Thornquist 1993</p> <p>McArthur et al.</p>

	responding to student errors.	1997
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<p>Building and Practicing Fact Families Through dialog and the use of manipulatives, tutees discover relationships between numbers. They create new fact families, record facts on flash cards and charts, and review facts from previous lessons.</p>	<p><i>Discovering relationships:</i> Assisting the learner to take responsibility for the construction of knowledge is one of the seven best strategies for enhancing mathematics education.</p> <p>Mathematics is no longer an instruction process in which the teacher transmits information to students; instead we must replace this format with one in which the students construct their own understandings.</p> <p><i>Manipulatives:</i> Hands-on tools help children begin to visualize and construct for themselves the patterns of our number system. These skills enable them to understand computation and develop efficient strategies for learning facts.</p> <p>Mathematics achievement increases when manipulatives are used.</p> <p>Although teaching a mathematics topic with concrete materials (or even pictures) may take longer, such materials do have a payoff in increased understanding and achievement.</p>	<p>Bickmore-Brand 1993</p> <p>D’Ambrosio, Johnson and Hobbs 2002</p> <p>Hamilton-Weiler 1988</p> <p>Suydam and Higgins 1977</p> <p>Suydam 1984</p>
<p>SOLVE the Problem Tutees solve word problems based on the reading passages.</p> <p>A heuristic called SOLVE prompts tutees to: Study the problem Organize the facts Line up a plan Verify the plan Examine results</p>	<p>The single best way to grow a better brain is to engage in challenging problem solving. Surprisingly, it doesn't matter if we come up with the right answer or not: the neural growth happens because of the process, not because we have found the correct answer.</p> <p>In classrooms with a problem-solving emphasis, children perceive mathematics as a problem-solving endeavor in which many different strategies are considered viable and communicating mathematical thinking is an integral part of the task....they recognize and accept a variety of solution strategies, with many of the children valuing all solutions equally and assuming a shared responsibility with the teacher for their mathematics learning. Children had varying perceptions of what it meant to succeed in mathematics, but success was not determined by speed and accuracy.</p> <p>A classroom environment that would support the learning of mathematics with meaning should have several characteristics: students feel comfortable making and correcting mistakes; rewards are given for sustained effort and progress, not the number of problems completed; and students think through and explain their solutions instead of seeking or trying to recollect the "right" answer or method.</p> <p>Through exploring, experimenting, trying out a hypothesis and finally, solving problems, children make learning personal and</p>	<p>Jensen 1998</p> <p>Franke 1997</p> <p>Cobb et al. 1988</p> <p>Piaget 1963</p>

	meaningful. Children understand only what they discover or invent themselves.	
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<p>SOLVE the Problem (continued)</p> <p>Calculator use is encouraged when computation interferes with the problem-solving process.</p>	<p>The process of problem solving--making choices and learning from them--is facilitated by teachers who observe, listen and ask open-ended questions that further the process: questions such as, "What will happen if...?" and "What other ways can you think of...?" Problem solving becomes a cycle of learning when mistakes are made and different solutions have to be tried. This discovery process allows children to construct their own learnings. Most problems have more than one solution; some problems cannot be solved. Experiences with these sorts of problems promote learning in young children.</p> <p>Thinking about thinking is important because children need a thinking vocabulary. During problem solving, the teacher (tutor) should think aloud so that students can follow demonstrated thinking processes...paired problem-solving is another useful strategy.</p> <p><i>Calculator use:</i> Students may miss the main objective of a lesson when they are caught in the trap of computation.</p>	<p>Britz 1993</p> <p>Blakey and Spence 1990</p> <p>Smith 1998</p>
<p>Write the Problem</p> <p>Tutees use the passages to form their own questions and write their own word problems.</p> <p>Tutors read a first draft of the tutee's problem and offer support and assistance as needed.</p>	<p>Writing their own mathematics problems ... is an important and effective way for students to see the relevance of mathematics.</p> <p>Students need a chance to assimilate information, make connections and face concepts that still confuse them. This kind of writing is a means of learning, a way into understanding through articulating.</p> <p>Writing is the ultimate act of synthesis.</p> <p>A safe environment becomes especially important when students are working on challenging mathematics and science and are expected to understand what they are learning. Open-ended problem solving and inquiry involve complex tasks that are more ambiguous and often mean more risks for students than traditional activities.</p> <p>Mathematical literacy requires an environment where students work together, observing and investigating the uses of numbers, asking questions and planning strategies to find the answers.</p>	<p>D'Ambrosio, Johnson and Hobbs 2002</p> <p>Hamilton-Weiler 1988</p> <p>Snow et al. 1998</p> <p>Henningsten and Stein 1997</p> <p>Whitin, Mills and O'Keefe 1990</p>

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<p>Tutee's Journal Writing Tutees end each lesson by reflecting and writing in journal pages. They answer questions that prompt them to recall and summarize new concepts, review previous lessons and evaluate their own progress.</p>	<p>Journals can promote fluency in reading and writing, encourage risk taking, provide opportunities for reflection and promote the development of written language conventions.</p> <p>Students need a chance to assimilate information, make connections and face concepts that still confuse them. This kind of writing is a means of learning, a way into understanding through articulating.</p> <p>Low-achieving math students using write-to-learn techniques improved state competency test results at a greater percentage than average math students in traditional classrooms.</p> <p>Writing to learn improves higher-order reasoning skills.</p>	<p>Routman 2000</p> <p>Hamilton-Weiler 1988</p> <p>Gladstone 1987</p> <p>Gere 1985, Barr and Healy 1988</p>
<p>Tutor debriefing: Writing and communication skills develop as tutors commit thoughts to a journal. Group sharing allows tutors to discuss successes and challenges. Debriefing allows coordinator to offer support and suggestions based on observations.</p>	<p><i>Debriefing:</i> Tutors need ongoing supervision and support. Younger tutors will require more structure and closer supervision. In periodic group meetings, older tutors gain psychological support by talking out frustrations and sharing success stories. Tutors can learn from each other's experiences as well as from staff suggestions for handling problems.</p> <p>The most effective tutoring programs provide on-going training and support for tutors.</p> <p><i>Writing:</i> Journals can promote fluency in reading and writing, encourage risk taking, provide opportunities for reflection and promote the development of written language conventions.</p>	<p>Gaustad 1993</p> <p>U.S. Dept. of Education 1997</p> <p>Routman 2000</p>

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